LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – STATISTICS

FIFTH SEMESTER – APRIL 2010

ST 5400 - APPLIED STOCHASTIC PROCESSES

Date & Time: 27/04/2010 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

PART – A

Answer ALL the questions **(10 x 2 = 20 marks)**

1. Give an example for a discrete time discrete state stochastic process.
2. Define covariance stationary process.
3. Give one example which (i) of two states communicate (ii) do not communicate.
4. Define Markov process.
5. Define stationary independent increments process.
6. Give an example of irreducible Markov chain.
7. Define a recurrent state.
8. Define an aperiodic state. with an example.
9. What is a Martingale?
10. Define the term: mean Recurrence Time.

PART – B

Answer any FIVE of the following: **(5 x 8 = 40 marks)**

11. Show that communication is an equivalence relation.
12. Show that the one step transition probability matrix of a Markov chain is a stochastic matrix.
13. State and prove Chapman – Kolmogorov equations.
14. Find the periodicities of the states of a Markov chain with one step transition probability matrix.

$$\begin{matrix} 0 \\ 1 \\ 2 \end{matrix}\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

15. Consider the following transition probability matrix $\begin{matrix} 0 \\ 1 \\ 2 \end{matrix}\begin{bmatrix} 0 & 1 & 1 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}.$

    using necessary and sufficient condition for recurrence, examine the nature of all the states.
16. Derive any one property of Poisson Process.
17. Explain the postulates of Poisson process.
18. Describe the weakly stationary and strongly stationary processes.

PART – C

Answer any TWO of the following **(2 x 20 = 40 marks)**

19. a) Let $\{Z_i, i = 1,2... \}$ be a sequence of independent random variables with means 1.

    Show that $X_n = \prod_{i=1}^{n} z_i$ is a Martingale.

    b) Show that every stochastic process with independent increments is a Markov process.

20. Suppose that the weather on any day depends on the weather conditions for the previous two days. To be exact, suppose that if it was sunny today and yesterday. then it will be sunny tomorrow with probability 0.8; if it was sunny today but cloudy yesterday, then it will be sunny tomorrow with probability 0.6; if it was cloudy today but sunny yesterday, then it will be sunny tomorrow with probability 0.4; if it was cloudy for the last two days, then it will be sunny tomorrow with probability 0.1. Transform the above model into a Markov chain and write down the TPM. Find the stationary distribution of the Markov chain.

    On what fraction of days in long run is it sunny?

21. Derive Yule – Furry process and find the expression for $P_x(t)$.

22. Write the short notes on the following:
    a) One dimensional random walk
    b) Periodic states
    c) Martingales

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